**Differentiating e^x Solutions**

1.

**a** $y = e^{6x}$

$\frac{dy}{dx} = 6e^{6x}$

**b** $y = e^{-\frac{1}{3}x}$

$\frac{dy}{dx} = -\frac{1}{3}e^{-\frac{1}{3}}$

**c** $y = 7e^{2x}$

$\frac{dy}{dx} = 2 \times 7e^{2x} = 14e^{2x}$

**d** $y = 5e^{0.4x}$

$\frac{dy}{dx} = 0.4 \times 5e^{0.4x} = 2e^{0.4x}$

**e** $y = e^{3x} + 2e^{x}$

$\frac{dy}{dx} = 3e^{3x} + 2e^{x}$

**f** $y = e^{x}(e^{x} + 1) = e^{2x} + e^{x}$

$\frac{dy}{dx} = 2e^{2x} + e^{x}$

2.

**a** $y = e^{3x}$

$\frac{dy}{dx} = 3e^{3x}$

When $x = 2$,

$\frac{dy}{dx} = 3e^{3 \times 2} = 3e^{6}$

**b** When $x = 0$,

$\frac{dy}{dx} = 3e^{3 \times 0} = 3$

**c** When $x = -0.5$,

$\frac{dy}{dx} = 3e^{3 \times -0.5} = 3e^{-1.5}$

3.

$f(x) = e^{0.2x}$

$f'(x) = 0.2e^{0.2x}$

The gradient of the tangent when $x = 5$

is $f'(5) = 0.2e^{0.2 \times 5} = 0.2e$

$f(5) = e^{0.2 \times 5} = e$

The equation of the tangent in the form

$y = mx + c$

is $e = 0.2e \times 5 + c$

$e = e + c$

so $c = 0$

Therefore the tangent to the curve at the point $(5, c)$ is in the form $y = mx$.

Thus it so goes through the origin.

4.

**a i** $e^{2} = 7.39$

**ii** $-e^{-2} = -0.14$

**b i** $\frac{y - e^{2}}{x - 2} = e^{2}$

$\Rightarrow y = e^{2}x - e^{2}$

**ii** $\frac{y - e^{-2}}{x - 2} = -e^{-2}$

$\Rightarrow y - e^{-2} = -e^{-2}x + 2e^{-2}$

$\Rightarrow y = \frac{3 - x}{e^{2}}$

5.

Tangent is

$\frac{y - e^{2}}{x - 4} = \frac{1}{2}e^{2}$

$\Rightarrow 2y - 2e^{2} = e^{2}x - 4e^{2}$

$\Rightarrow y = \frac{1}{2}e^{2}x - e^{2}$

When $x = 6$, $y = 2e^{2}$

Point is $(6, 2e^{2})$

6.

When $x = 1$, gradient of tangent $= 2e^2$

So gradient of normal $= -\frac{1}{2e^2}$

Equation of normal $\frac{y - e^2}{x - 1} = -\frac{1}{2e^2}$

$\Rightarrow 2e^2 y - 2e^4 = -x + 1$

Cuts $x$-axis when $y = 0$

$\Rightarrow -2e^4 = -x + 1$

$x = 2e^4 + 1 \approx 110$

Point on $x$-axis is $(2e^4 + 1, 0)$

7.

**a** $-2e^t$

**c** $e^t + 5t^4$

**d** $\frac{3}{2}t^{\frac{1}{2}} + 2e^t$

**h** $14t - 2 + 4e^t$

8.

**a** $f'(x) = 3 + e^x$

$f'(0) = 3 + 1 = 4$

**d** $f'(x) = 5e^x - 2x^{-3}$

$f'(-\frac{1}{2}) = 5e^{-\frac{1}{2}} + 16$ [ 19.0 (3sf)

9.

**a** $\frac{dy}{dx} = e^x - 2$

SP: $e^x - 2 = 0$

$x = \ln 2$

$\frac{d^2y}{dx^2} = e^x$

$x = \ln 2$: $\frac{d^2y}{dx^2} = 2$

$\therefore (\ln 2, 2 - 2\ln 2)$, min

**d** $\frac{dy}{dx} = 4 - 5e^x$

SP: $4 - 5e^x = 0$

$x = \ln \frac{4}{5}$

$\frac{d^2y}{dx^2} = -5e^x$

$x = \ln \frac{4}{5}$: $\frac{d^2y}{dx^2} = -4$

$\therefore (\ln \frac{4}{5}, 4\ln \frac{4}{5} - 4)$, max

10.

$\frac{dy}{dx} = 1 + ke^x$

$\frac{d^2y}{dx^2} = ke^x$

$\therefore (1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y$

$= (1 - x)ke^x + x(1 + ke^x) - (x + ke^x)$

$= ke^x - kxe^x + x + kxe^x - x - ke^x = 0$

11.

**a** $x = 0 \ \therefore y = \frac{1}{10}$

$\frac{dy}{dx} = \frac{2}{5} + \frac{1}{10}e^x$, grad $= \frac{1}{2}$

$\therefore$ grad of normal $= -2$

$\therefore y = -2x + \frac{1}{10}$

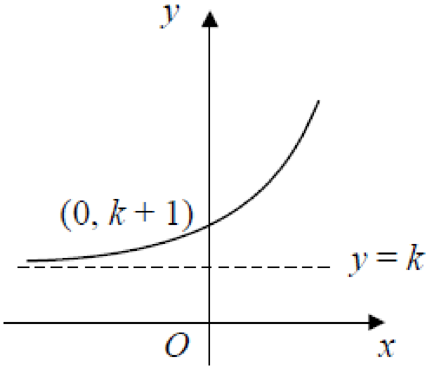
$20x + 10y - 1 = 0$

**b** $y = 0 \ \therefore x = \frac{1}{20}$

$(\frac{1}{20}, 0)$

12.

**a**




**b** $x = 2 \quad \therefore y = e^2 + k$

$\frac{dy}{dx} = e^x, \quad \text{grad} = e^2$

$\therefore \; y - (e^2 + k) = e^2(x - 2)$

$[\, y = e^2x - e^2 + k \,]$

**c** $(-1, 0) \quad \therefore \quad 0 = -e^2 - e^2 + k$

$k = 2e^2$

13.

**a** $\frac{dy}{dx} = e^x, \quad \text{grad at } P = e^p$

tangent: $\quad y - e^p = e^p(x - p)$

$(0, 0) \quad \therefore \quad 0 - e^p = e^p(0 - p)$

$e^p(p - 1) = 0$

$e^p \neq 0 \quad \therefore p = 1$

**b** $P\,(1, e), \quad \text{grad at } P = e$

$\therefore \; \text{grad of normal} = -\frac{1}{e}$

$\therefore \; y - e = -\frac{1}{e}(x - 1)$

at $Q, \; y = 0 \quad \therefore x = e^2 + 1$

$\therefore \; \text{area} = \frac{1}{2} \times (e^2 + 1) \times e = \frac{1}{2}e(1 + e^2)$

14.

(i) $N = 50e^{0.1t}$

When $N = 200$

$200 = 50e^{0.1t}$

$e^{0.1t} = 4$

$0.1t = \ln 4$

$t = 13.86$

The population is greater than 200 after 14 weeks.

(ii) $\frac{dN}{dt} = 50 \times 0.1e^{0.1t} = 5e^{0.1t}$

When $t = 5$, $\frac{dN}{dt} = 5e^{0.5} = 8.24$

Rate of increase after 5 weeks is 8.24 mice / week.

(iii) $N = 50e^{0.1t} \quad \Rightarrow e^{0.1t} = \frac{N}{50}$

$\frac{dN}{dt} = 5 \times \frac{N}{50}$

$\frac{dN}{dt} = \frac{N}{10}$

The value of k is 0.1.

(iv) When $N = 200$, $\frac{dN}{dt} = \frac{200}{10} = 20$

The rate of increase is 20 mice / week.

(v) Resources such as food and space are unlikely to be able to sustain the population as it becomes very large. Also, the model takes no account of mice dying.